We being with considering subsets of the *everything/nothing* object E, defined as the set of all infinite length binary strings $[0,1]^\infty$.

A conscious observer of these strings will only examine a finite number of bits prior to making a decision on the meaning. An observer moment will therefore be an infinite subset of E of all strings that match that particular observer moments details, according to the observer.

An atomic OM is the subset defined by fixed finite length prefix, followed by an infinite string of "don't care" bits. We can write these like

$$1011*\ldots$$

where the prefix is the 4 bits 1011, followed by "don't care" bits represented by *. The subset this represents is all bitstrings that start with the 4 bits 1011.

We can index these by the natural numbers, eg:

$$\begin{array}{lcr} 101*\ldots & = & A_{13} \\ 0*\ldots & = & A_2 \\ *\ldots & = & A_1 = E \end{array}$$

The idea is that the leading 1 bit of the number is shorn off, then take the remaining bits in the binary expansion of the index.

An observer moment can match a number of different prefixes, corresponding to equivalent descriptions of the OM. As such, an OM will be represented by discrete union (possibly infinite) of atomic OMs. For example, to insert a "don't care" bit into the prefix, a union of two atomic OMs can be used:

$$10*01*\ldots \quad = \quad A_{49} \cup A_{53} \tag{1}$$

Let's call the set of all discrete unions of atomic OMs the set $\mathcal{O} \subseteq 2^E$, ie $\forall x \in \mathcal{O}, \exists I \subset \mathbb{N} : x = \bigcup_{i\in I} A_i$, which is a superset of the set of all observer moments.

I had an insight in the late noughties after publishing my book that information satisfies some sort of triangle equality, ie if two pieces of information are disjoint, then the information is additive, like say the information of a particle's x coordinate, and of it's y coordinate adds up to twice that of just the x coordinate alone, but if the information is about overlapping desiderata, and in particular contradictory, then less information is available in the combined situation. In the limit of the everything/nothing object, there is precisely zero information.

When information is additive, each successive measurement refines the outcome, which can be modelled via set intersection. If our OM initially is A, then a measurement outcome B will restrict the OM to $A \cap B$. Conversively, if the information is contradictory, the outcome will be $A \cup B$.

The triangle inequality nature of information is suggestive of a vector space. An important property we shall see later is that the sum of the vectors corresponding to two OMs will be given by the sum of the vectors of the union and intersection, ie vector summation corresponds to the process of gaining information about the environment.

Let us consider the following mapping of the bitstring to a vector: $1 \to \frac{1}{2^n}; 0 \to -\frac{1}{2^n}; * \to 0$, where n is the number of fixed bits in the bitstring, and denote the vector representation of a subset X as $\mathcal{V}^R_X$. For example, eq (1) can be expressed as:

$$\begin{array}{lcr} 10*01*\ldots & = & 10001*\ldots \cup 10101*\ldots \\ V_{A_{49}\cup A_{53}} & = & V^R_{A_{49}} + V^R_{A_{53}} \\ [\frac{1}{16}, -\frac{1}{16}, 0, -\frac{1}{16}, \frac{1}{16}, 0, \ldots] & = & [\frac{1}{32}, -\frac{1}{32}, -\frac{1}{32}, -\frac{1}{32}, \frac{1}{32}, 0, \ldots] + [\frac{1}{32}, -\frac{1}{32}, \frac{1}{32}, -\frac{1}{32}, \frac{1}{32}, 0, \ldots] \end{array}$$

It is not too hard to see that for disjoint sets A and B such that $A \cap B = \emptyset$, $V^R_{A\cup B} = V^R_A + V_B$.

But there is a problem, which can be most keenly seen by considering the sets

$$\begin{array}{rcl} B_0 & = & A_5 \cup A_6 = 01*\ldots \cup 10*\ldots \\ B_1 & = & A_4 \cup A_7 = 00*\ldots \cup 11*\ldots \end{array}$$

These two sets partition the everything E, ie $E = B_0 \cup B_1$, and $B_0 \cap B_1 = \emptyset$. Yet according to this map, $V^R_{B_0} = V^R_{B_1} = 0$. So the V^R vector space is not isomorphic to the set $\mathcal{O}$.

The partitions of E $\{B_0, B_1\}$ and $\{A_2, A_3\}$ are naturally orthogonal to each other. If I know which partition out of B_0 and B_1 I am presently located, I have no information at all about which of A_2 or A_3 I might find myself in. And vice versa. This is strongly suggestive of a Heisenberg uncertainty relationship. So the idea is to embed each bit in a two dimensional space, and rotate the leading component of the by right angles depending on whether the previous bit is extending into the positive or negative plane. We can use the complex numbers to represent this 2D space, for arithmetical convenience.

The precise mapping of V_{A_j} is as follows:

Let

$$E_j = \begin{cases} -1 & j \bmod 4 = 0 \\ 1 & j \bmod 4 = 1 \\ -i & j \bmod 4 = 2 \\ i & j \bmod 4 = 3 \end{cases} \tag{2}$$

and let

$$\mathrm{pfix}(j) = \lfloor j 2^{-L(j)} \rfloor \tag{3}$$

where $L(j) = \lfloor \log_2 j \rfloor$, and let e_j be the usual cartesian unit vector $[\delta_{1j}, \delta_{2j}, \ldots]$

Then

$$V_{A_j} = \frac{1}{2} V_{\mathrm{pfix(j)}} + 2^{-L(j)} E_j e_{L(j)} \tag{4}$$

Now we have $V_{B_0} = V_{A_5} + V_{A_6} = [0, \frac{1-i}{4}, 0, \ldots]$ and $V_{B_1} = V_{A_4} + V_{A_7} = [0, \frac{i-1}{4}]$.

Lemma 1 $\forall i, j, k : A_i = A_j \cup A_k,\ V_{A_i} = V_{A_j} + V_{A_k}$.

Proof: If $A_i = A_j \cup A_k$, then $|j-k| = 1$ and $i = \lfloor j/2 \rfloor$, ie the prefixes of A_j and A_k are just the prefix of A_i with either a 0 or 1 appended. So, $\mathrm{pfix}(j) = \mathrm{pfix}(k) = i$, $L(j) = L(k)$ and $E_j = -E_k$. Then, from (4), $V_{A_j} + V_{A_k} = V_{A_i}$.

$\square$

Let's extend V to $\mathcal{O}$ by expanding each element $X \in \mathcal{O}$ as a union of a disjoint set of atomic OMs $X = \bigcup_{i \in x \subseteq \mathbb{N}} A_i$, ie $\forall i, j \in x, A_i \cap A_j = \emptyset$. Let $V_X = \sum_{i \in x} V_{A_i}$. Even though x is not uniquely defined, Lemma 1 guarantees that V_X is uniquely defined.

Theorem 1

$$V_X + V_Y = V_{X \cup Y} + V_{X \cap Y}$$

Proof: If $X \subseteq Y$, then $X \cup Y = Y$ and $X \cap Y = X$ and the theorem is trivially true. If neither set is a subset of the other, then define $X' = X - X \cap Y$ and $Y' = Y - X \cap Y$. The sets X', Y, and $X \cap Y$ are mutually disjoint. Therefore

$$\begin{aligned} V_{X \cup Y} &= V_{X' \cup Y' \cup X \cap Y} \\ &= V_{X'} + V_{Y'} + V_{X \cap Y} \\ &= V_X - V_{X \cap Y} + V_{Y'} - V_{X \cap Y} + V_{X \cap Y} \\ &= V_X + V_Y - V_{X \cap Y} \end{aligned}$$

$\square$

Conjecture 1 *The set of all vectors $\{\alpha V_X : X = \mathcal{O}, \alpha \in \mathbb{C}\}$ forms a complete vector space.*