

$$\begin{aligned}
 \nabla \mathbf{f} &= \mathbf{x} \frac{\partial f_1}{\partial x} \mathbf{x} + \mathbf{y} \frac{\partial f_2}{\partial x} \mathbf{x} + \mathbf{x} \frac{\partial f_1}{\partial y} \mathbf{y} + \mathbf{y} \frac{\partial f_2}{\partial y} \mathbf{y} \\
 &= \mathbf{x} \left(\frac{\partial f_1}{\partial x} \mathbf{x} + \frac{\partial f_1}{\partial y} \mathbf{y} \right) + \mathbf{y} \left(\frac{\partial f_2}{\partial x} \mathbf{x} + \frac{\partial f_2}{\partial y} \mathbf{y} \right) \\
 &= \mathbf{x}(\nabla f_1) + \mathbf{y}(\nabla f_2)
 \end{aligned}$$