Seminar in functional analyses

Consider all spaces to be of finite dimension.

Ideal norm :

Let α be a norm.

1. if α operates on $L(E,F)$ for every E & F, and
2. For ever operator of rank 1 (that has a form $x^* \otimes y \in L(E,F)$) $\alpha(x^* \otimes y) = \|x^*\| \cdot \|y\|$, and
3. $\forall E,F,G,H$ Banach spaces and $E \xrightarrow{S} F \xrightarrow{T} G \xrightarrow{V} H$ linear operators $\alpha(VTS) \le \|V\| \alpha(T) \|S\|$

then α is called "Ideal norm".

For example :

the regular norm,

p-summing norm : $\Pi_p(T) = \inf_c \left\{ \forall \{x_i\} \left(\sum_{i=1}^n \|Tx_i\|^p \right)^{1/p} \le c \cdot \sup_{\|x^*\| \le 1} \left[\sum (x^*(x_i))^p \right]^{1/p} \right\}$

Note that $\Pi_p(T) \xrightarrow[p \to \infty]{} \Pi_\infty(T) = \|T\|$ since the left side of the inequality goes to $\max \|Tx_i\|$ and the right side (Han-Banach) to $c \cdot \max \|x_i\|$.

q-nuclear norm : $\nu_q(T) = \inf_{l-decompositions} \{\|B\| \cdot \|\lambda\| \cdot \|A\|\}$

$$\begin{array}{ccc} X & \xrightarrow{T} & Y \\ A \downarrow & & \uparrow B \\ l_\infty^n & \xrightarrow[\lambda]{} & l_q^n \end{array}$$

where l-decompositions are $T = B \circ \lambda \circ A$ where $\lambda(e_i) = \lambda_i \cdot e_i$ (diagonal).

q'-nuclear norm : $\tilde{\nu}_q(T) = \inf_{tensor-decomposition} \left\{ \left(\sum \|x_i^*\|^q \right)^{1/q} \right\} \cdot \sup_{\|y^*\|=1} \left\{ \left[\sum (y^*(y_i))^{q^*} \right]^{1/q^*} \right\}$

where $\frac{1}{q} + \frac{1}{q^*} = 1$ and tensor-decompositions are of the form :

$T = \sum (x_i^* \otimes y_i) \Leftrightarrow Tf = \sum \langle x_i^*, y_i \rangle f$

Theorem : $\tilde{\nu}_q(T) \equiv \nu_q(T)$

Proof :

Consider $A^* : l_1^n \to X^*$ and define $z_i^* = A^* e_i$. We get $\|A\| = \|A^*\| = \max_i \{\|z\|_i^*\}$

(notice that $A = \sum_{i=1}^n z_i^* \otimes e_i \Leftrightarrow A^* = \sum_{i=1}^n e_i \otimes z_i^*$).