Powers of Convolution of Gate Function h

$$h = \left\{ \begin{array}{c} x < -\frac{1}{2} : 0 \\ -\frac{1}{2} < x \leq \frac{1}{2} : 1 \\ x \geq \frac{1}{2} : 0 \end{array} \right\} \tag{1}$$

$$h^{*2} = \left\{ \begin{array}{c} x < -1 : 0 \\ -1 < x \leq 0 : x + 1 \\ 0 < x \leq 1 : 1 - x \\ x \geq 1 : 0 \end{array} \right\} \tag{2}$$

$$h^{*3} = \left\{ \begin{array}{c} x < -\frac{3}{2} : 0 \\ -\frac{3}{2} < x \leq -\frac{1}{2} : \frac{x^2}{2} + \frac{3x}{2} + \frac{9}{8} \\ -\frac{1}{2} < x \leq \frac{1}{2} : \frac{3}{4} - x^2 \\ \frac{1}{2} < x \leq \frac{3}{2} : \frac{x^2}{2} - \frac{3x}{2} + \frac{9}{8} \\ x \geq \frac{3}{2} : 0 \end{array} \right\} \tag{3}$$

$$h^{*4} = \left\{ \begin{array}{c} x < -2 : 0 \\ -2 < x \leq -1 : \frac{x^3}{6} + x^2 + 2x + \frac{4}{3} \\ -1 < x \leq 0 : -\frac{x^3}{2} - x^2 + \frac{2}{3} \\ 0 < x \leq 1 : \frac{x^3}{2} - x^2 + \frac{2}{3} \\ 1 < x \leq 2 : -\frac{x^3}{6} + x^2 - 2x + \frac{4}{3} \\ x \geq 2 : 0 \end{array} \right\} \tag{4}$$

$$h^{*5} = \left\{ \begin{array}{c} x < -\frac{5}{2} : 0 \\ -\frac{5}{2} < x \leq -\frac{3}{2} : \frac{x^4}{24} + \frac{5x^3}{12} + \frac{25x^2}{16} + \frac{125x}{48} + \frac{625}{384} \\ -\frac{3}{2} < x \leq -\frac{1}{2} : -\frac{x^4}{6} - \frac{5x^3}{6} - \frac{5x^2}{4} - \frac{5x}{24} + \frac{55}{96} \\ -\frac{1}{2} < x \leq \frac{1}{2} : \frac{x^4}{4} - \frac{5x^2}{8} + \frac{115}{192} \\ \frac{1}{2} < x \leq \frac{3}{2} : -\frac{x^4}{6} + \frac{5x^3}{6} - \frac{5x^2}{4} + \frac{5x}{24} + \frac{55}{96} \\ \frac{3}{2} < x \leq \frac{5}{2} : \frac{x^4}{24} - \frac{5x^3}{12} + \frac{25x^2}{16} - \frac{125x}{48} + \frac{625}{384} \\ x \geq \frac{5}{2} : 0 \end{array} \right\} \tag{5}$$